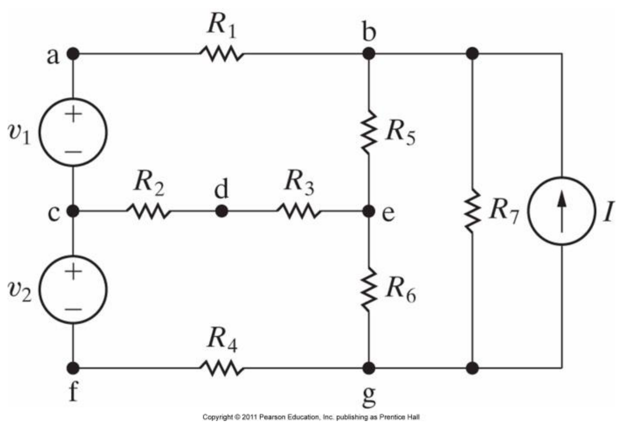
4.1 Terminology

TABLE 4.1 Terms for Describing Circuits

Name	Definition	Example From Fig. 4.3
node	A point where two or more circuit elements join	a
essential node	A node where three or more circuit elements join	b
path	A trace of adjoining basic elements with no elements included more than once	$v_1 - R_1 - R_5 - R_6$
branch	A path that connects two nodes	R_1
essential branch	A path which connects two essential nodes without passing through an essential node	$v_1 - R_1$
loop	A path whose last node is the same as the starting node	$v_1 - R_1 - R_5 - R_6 - R_4 - v_2$
mesh	A loop that does not enclose any other loops	$v_1 - R_1 - R_5 - R_3 - R_2$
planar circuit	A circuit that can be drawn on a plane with no crossing branches	Fig. 4.3 is a planar circuit Fig. 4.2 is a nonplanar circuit

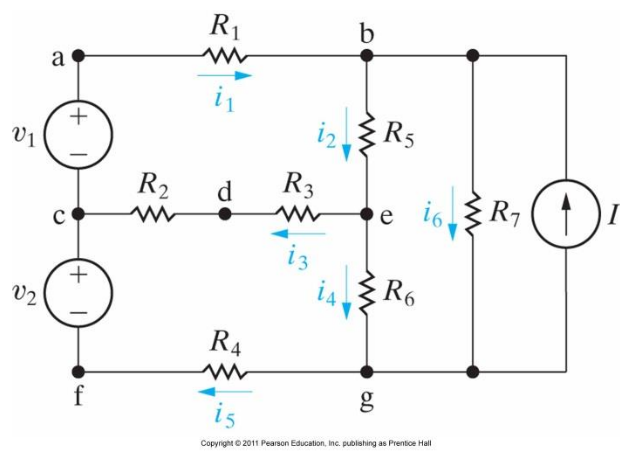
Example 4.1



a. Nodes: **a, b, c, d, e, f, g**
b. Essential Nodes: **b, c, e, g**
c. Branches: **v_1, v_2, R_1, R_2, R_3, R_4, R_5, R_6, R_7, I**
d. Essential Branch: **v_1-R_1, R_2-R_3, v_2-R_4, R_5, R_6, R_7, I**
e. Meshes: **v_1-R_1-R_5-R_3-R_2, : v_2-R_2-R_3-R_6-R_4, R_5-R_7-R_6, R_7-I**

Determining the simultaneous equations

Number of unknown currents equals the number of branches (b); given n nodes then there are n-1 possible equations from the KCL. To get the remaining equations need to use KVL: b-(n-1)



Applying this to essential branches (b_e) and nodes (n_e):

Number of KCL equations = n_e-1
Number of KVL equations = b_e-(n_e-1)

From above n_e = 4 and b_e = 6

Therefor there are 3 KCL and 3 KVL equations needed

KCL equations:

$$-i_1 + i_2 + i_6 - I = 0$$

$$i_1 - i_3 - i_5 = 0$$

$$i_3 + i_4 - i_2 = 0$$

KVL equations:

$$R_1 i_1 + R_5 i_2 + i_3(R_2 + R_3) - v_1 = 0$$

$$-i_3(R_2 + R_3) + R_6 i_4 + R_4 i_5 - v_2 = 0$$

$$-R_5 i_2 + R_7 i_6 - R_6 i_4 = 0$$

Rewriting

$$-i_1 + i_2 + 0i_3 + 0i_4 + 0i_5 + i_6 = I$$

$$i_1 + 0i_2 - i_3 + 0i_4 - i_5 + 0i_6 = 0$$

$$0i_1 - i_2 + i_3 + i_4 + 0i_5 + 0i_6 = 0$$

$$R_1 i_1 + R_5 i_2 + (R_2 + R_3)i_3 + 0i_4 + 0i_5 + 0i_6 = v_1$$

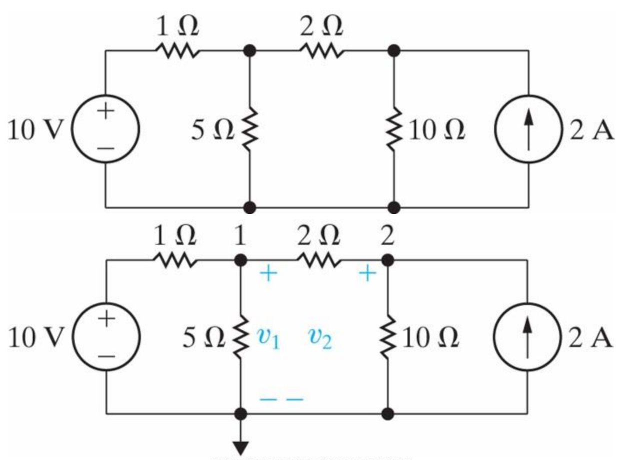
$$0i_1 + 0i_2 - (R_2 + R_3)i_3 + R_6 i_4 + R_4 i_5 + 0i_6 = v_2$$

$$0i_1 - R_5 i_2 + 0i_3 - R_6 i_4 + 0i_5 + R_7 i_6 = 0$$

If new variables are introduced we can actual solve with either:

n_e-1 node voltages OR b_e-(n_e-1) mesh currents

4.2 Introduction to the Node-Voltage Method



Node voltage: voltage rise from the reference node to the non-reference node.

Steps for Node-Voltage Method

1. Select one of the essential nodes as the reference node. (usually with most branches)
2. Define the appropriate node voltages
3. Now write the node voltage equations
   a. Write the current leaving each branch connected to a non-reference node as a function of node voltage.
   b. Sum the currents and set equal to zero as per KCL

i.e. @ node 1

$$\frac{v_1 - 10}{1} + \frac{v_1 - 0}{5} + \frac{v_1 - v_2}{2} = 0$$

@ node 2

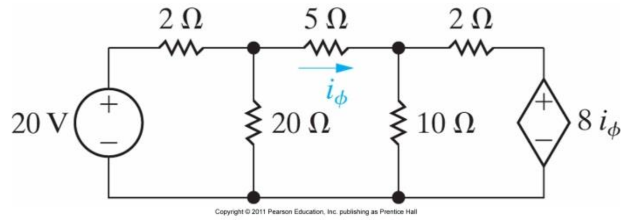
$$\frac{v_2 - v_1}{2} + \frac{v_2 - 0}{10} - 2 = 0$$

Solve for the unknown voltages; use that information to solve for the remaining unknowns

Review Example 4.2 and Assessment Problems 4.1 & 4.2

4.3 The Node-Voltage Method and Dependent Sources

Example 4.3



$$\frac{v_1-20}{2}+\frac{v_1}{20}+\frac{v_1-v_2}{5}=0$$

$$\frac{v_2-v_1}{5}+\frac{v_2}{10}+\frac{v_2-8i_\phi}{2}=0 \quad \text{where} \quad i_\phi=\frac{v_1-v_2}{5}$$

Substituting and solving

$$2v_2-2v_1+v_2+5v_2-40i_\phi=0 \quad \rightarrow \quad 8v_2-2v_1-40i_\phi=0$$

$$8v_2-2v_1-40\left(\frac{v_1-v_2}{5}\right)=0 \;\rightarrow\; 16v_2-10v_1=0 \rightarrow \;\; \boldsymbol{v_1=1.6v_2}$$

Again

$$10v_1-200+v_1+4v_1-4\left(\frac{10}{16}v_1\right)=0 \;\rightarrow v_1=16V \;\; and \;\; v_2=10V$$

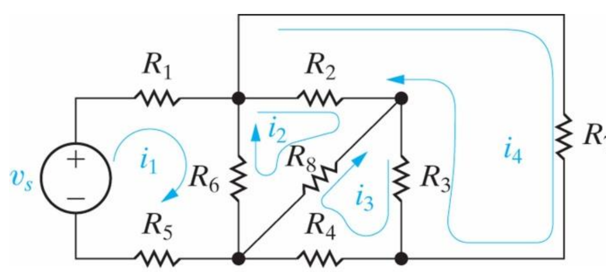
$$i_\phi=\frac{16-10}{5}=1.2A \;\; and \;\; p=i^2R=(1.2^2)5=7.2\text{W}$$

Review Assessment Problem 4.3

4.4 The Node-Voltage Method: Some Special Cases

Read over the material and examples for information on shortcuts and tricks: Assessment Problems 4.4 - 4.6 *(Material not explicitly covered on test)*

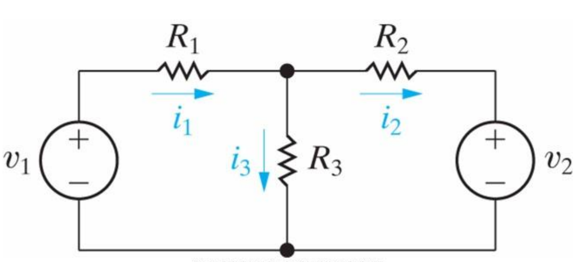
4.5 Introduction to the Mesh-Current Method

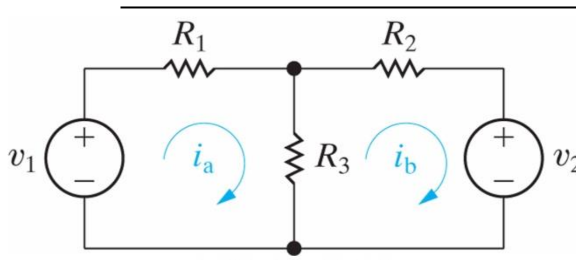


<u>Mesh:</u> a loop with no other loops inside it

<u>Mesh Current:</u> current that exists only in the perimeter of the mesh

Solve by writing: b_e-(n_e-1) mesh current equations (arrow indicates direction)





KCL and KVL Method:

$$i_1=i_2+i_3; \qquad v_1=i_1R_1+i_3R_3$$

$$-v_2=i_2R_2-i_3R_3$$

Mesh Method (n_e=2; b_e=3)

$$v_1=i_aR_1+(i_a-i_b)R_3$$

$$-v_2=(i_b-i_a)R_3+i_bR_2$$

Review Example 4.4 and Assessment Problem 4.7

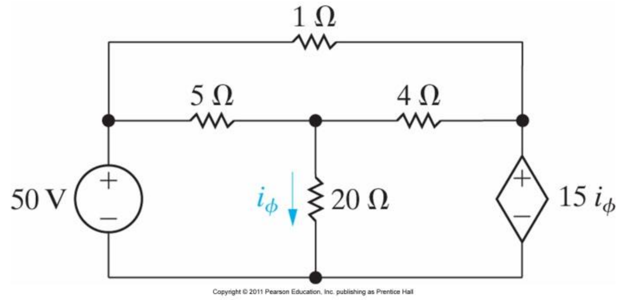
4.6 The Mesh-Current Method and Dependent Sources

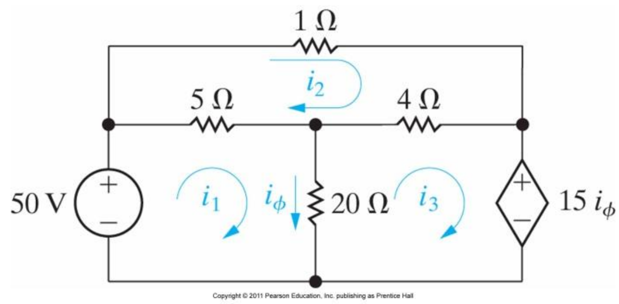
If $n_e = 4$ and $b_e = 6$ then 3 mesh equations are needed

$$5(i_1 - i_2) + 20(i_1 - i_3) = 50$$

$$i_2 + 4(i_2 - i_3) + 5(i_2 - i_1) = 0$$

$$4(i_3 - i_2) + 20(i_3 - i_1) + 15 i_\phi = 0$$





Where

$$i_\phi = i_1 - i_3$$

Rewriting

$$25 i_1 + 20 i_3 = 50$$

$$-5 i_1 + 10 i_2 - 4 i_3 = 0$$

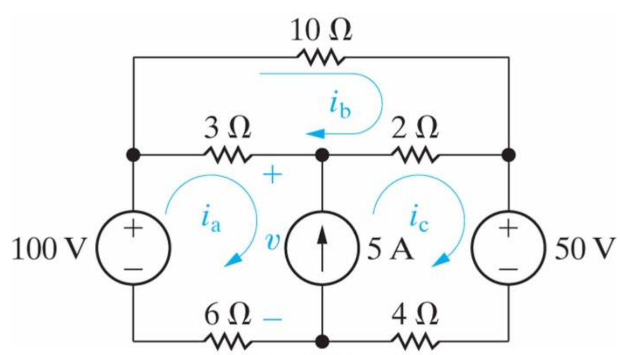
$$-5 i_1 - 4 i_2 + 9 i_3 = 0$$

Review Example 4.5 and Assessment Problems 4.8 - 4.9

4.7 The Mesh-Current Method: Some Special Cases

Current Source in a mesh: ($n_e = 4$; $b_e = 5$ then 2 mesh?)

Actually need three because the voltage across the current source is unknown *v.*



$$3(i_a - i_b) + v + 6 i_a = 100$$

$$10 i_b + 2(i_b - i_c) + 3(i_b - i_a) = 0$$

$$2(i_c - i_b) + 4 i_c - v = -50$$

Also, from the source:

$$i_c - i_a = 5$$

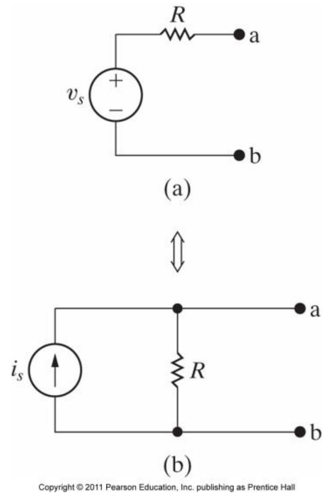
Read over the remaining material and examples for additional information: Assessment Problems 4.10 - 4.12

(Additional material not explicitly covered on test)

4.8 The Node-Voltage Method Versus the Mesh-Current Method

- Can you easily determine that one or the other will yield fewer simultaneous equations?
- Can the use of a super-node or super-mesh be utilized?

4.9 **Source Transformations**



Source Transformation: A simplification technique that allows for a voltage source in series with a resistor to be replaced with a current source in parallel with the same resistor and vice versa.

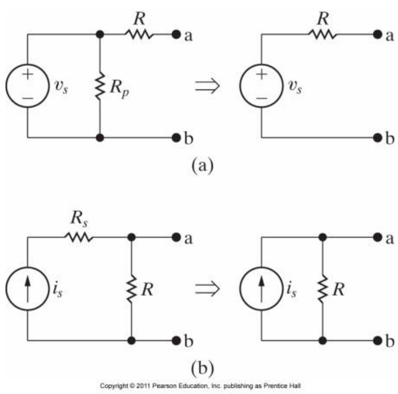
Assuming a load is connect across each circuit to the right and using Ohm's Law

$I_L = \frac{v_s}{R+R_L}$ for the voltage-series resistance circuit

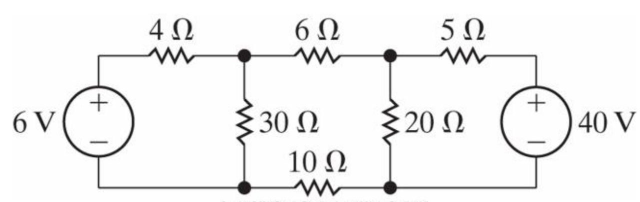
$I_L = \frac{R}{R+R_L} i_s$ for the current-parallel resistance circuit

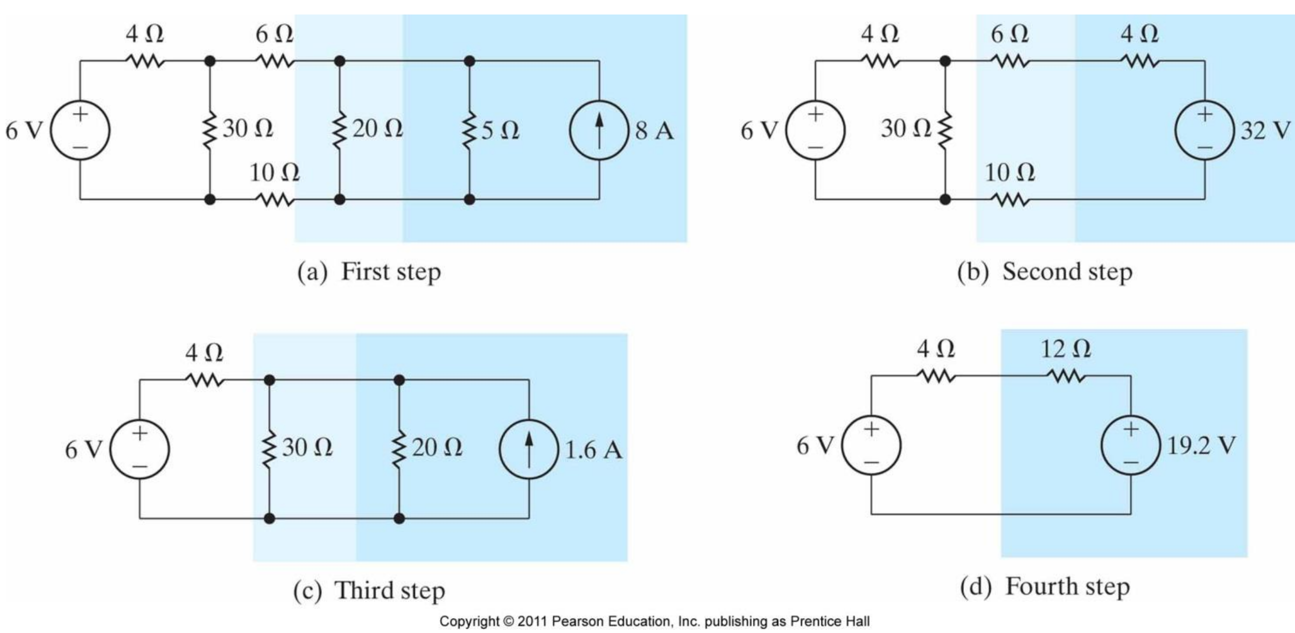
Setting the two equal will provide the relationship between them

$$I_L = \frac{v_s}{R+R_L} = \frac{R}{R+R_L} i_s \quad \rightarrow \quad \boldsymbol{i_s} = \frac{\boldsymbol{v_s}}{\boldsymbol{R}}$$



Note: The presence of a parallel resistance in the series voltage circuit or a series resistance in the parallel current circuit will not have an effect as seen at terminals a and b as the equivalent circuit depicts to the left. Review of *Example 4.8*

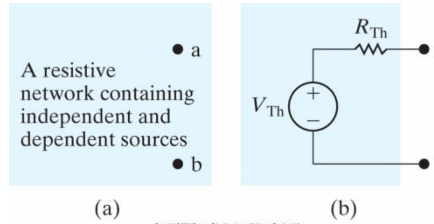




Review Example 4.9 and Assessment Problem 4.15

4.10 Thevenin and Norton Equivalents

<u>Thevenin equivalent circuit:</u> is an independent voltage source V_{Th} and series resistance R_{Th}, which replaces an interconnection of sources and resistors and is equivalent to the original circuit as seen at the terminals a and b.



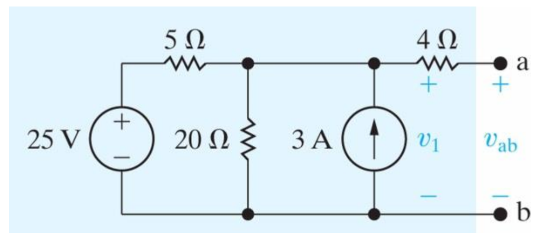
V_{Th} is the open circuit voltage as seen at the terminals a and b

Shorting terminals a and b together allows us to determine R_{Th} from Ohm's Law

$$R_{Th} = \frac{V_{Th}}{i_{sc}}$$

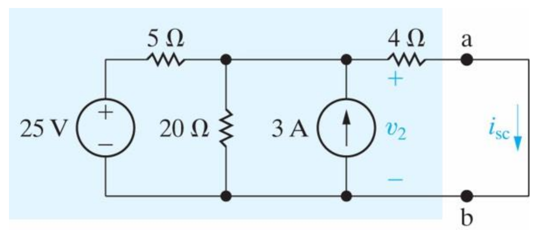
Finding a Thevenin Equivalent

Solving for the voltage requires analyzing with terminals a and b open. $(v_{ab} = V_{Th} = v_1)$



$$\frac{v_1 - 25}{5} + \frac{v_1}{20} - 3 = 0 \;\rightarrow\; v_1 = 32V = V_{Th}$$

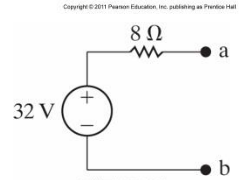
Shorting a and b allows us to determine i_{sc}



$$\frac{v_2 - 25}{5} + \frac{v_2}{20} - 3 + \frac{v_2}{4} = 0 \rightarrow \; v_2 = 16V$$

$$i_{sc} = \frac{v_2}{4} = 4A$$

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = 8\Omega$$

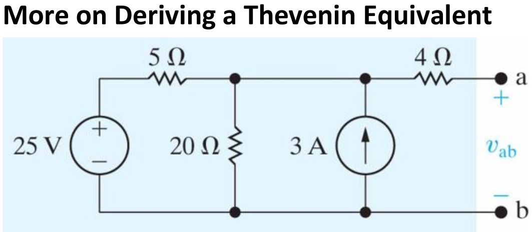


<u>Norton Equivalent Circuit:</u> an independent current source in parallel with a Norton equivalent resistance; can be derived from the Thevenin by use of a source transformation.

Note: It may be possible to derive both the Thevenin and Norton equivalents by directly taking source transformations of the original circuit.

Review Example 4.10 and Assessment Problems 4.16 - 4.18

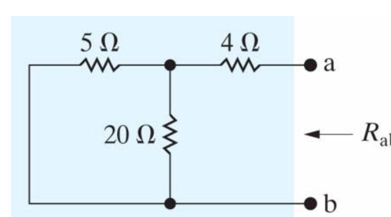
4.11 More on Deriving a Thevenin Equivalent



Calculating R_{Th} with **independent** sources.

Open current sources

Short voltage sources



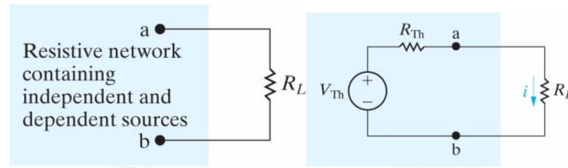
Find the equivalent resistance as seen at terminals a and b.

$$R_{Th} = 4 + \frac{5 * 20}{5 + 20} = 8\Omega$$

4.12 Maximum Power Transfer

Determine the value of R_L for maximum power transfer:

First replace the circuit with its Thevenin Equivalent Circuit



$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

Taking the derivative of p with respect to R_L.

$$\frac{dp}{dR_L} = {V_{Th}}^2 \left[\frac{(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right]$$

Setting the equation equal to zero for maximum power

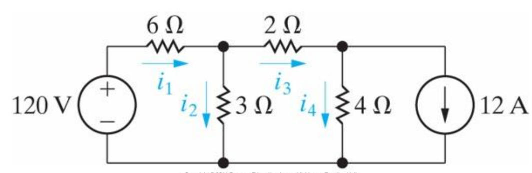
$$(R_{Th} + R_L)^2 - 2R_L(R_{Th} + R_L) = 0 \quad \rightarrow \quad R_L = R_{Th}$$

Thus

$$p_{max} = \frac{{V_{Th}}^2 R_L}{(2R_L)^2} = \frac{{V_{Th}}^2}{4R_L}$$

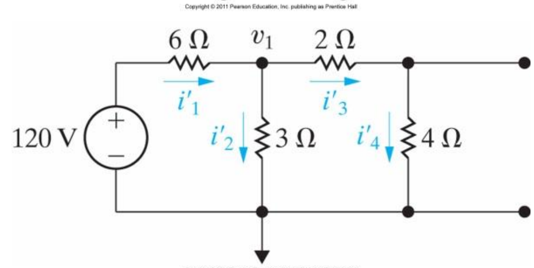
Review Example 4.12 and Assessment Problems 4.21 – 4.22

4.13 Superposition



<u>Superposition</u>: whenever a linear system is excited by more than one independent source the total response is the sum of the individual responses.

Finding the branch currents for the original figure.



1. Solve for the branch currents due to the 120V source; priming the current values. (Remove all other sources)

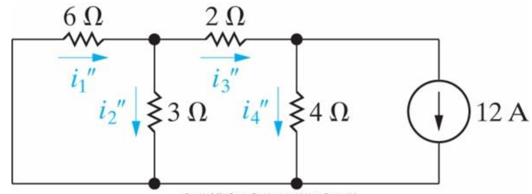
Nodal analysis about v_1

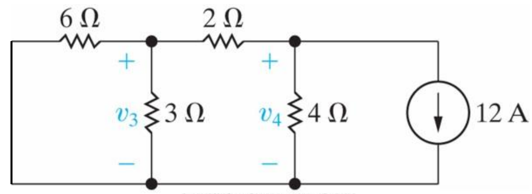
$$\frac{v_1 - 120}{6} + \frac{v_1}{3} + \frac{v_1}{2 + 4} = 0 \quad \rightarrow \quad v_1 = 30V$$

Solving for the currents

$$i'_1 = \frac{120 - v_1}{6} = 15A \qquad i'_2 = \frac{v_1}{3} = 10A \qquad i'_3 = i'_4 = \frac{v_1}{6} = 5A$$

2. Solve for the branch currents due to the 12A source; double priming the current values. (Remove all other sources)





Nodal analysis about 2 nodes

$$\frac{v_3}{6} + \frac{v_3}{3} + \frac{v_3 - v_4}{2} = 0$$

$$\frac{v_4 - v_3}{2} + \frac{v_4}{4} + 12 = 0$$

$$v_3 = -12V \qquad \& \qquad v_4 = -24V$$

Solving for the currents

$$i_1'' = \frac{-v_3}{6} = 2A \qquad i_2'' = \frac{v_3}{3} = -4A \qquad i_3' = \frac{v_3 - v_4}{2} = 6A \qquad i_4' = \frac{v_4}{4} = -6A$$

Finding the original branch currents

$$i_1 = i_1' + i_1'' = 15 + 2 = 17A$$

$$i_2 = i_2' + i_2'' = 10 - 4 = 6A$$

$$i_3 = i_3' + i_3'' = 5 + 6 = 11A$$

$$i_4 = i_4' + i_4'' = 5 - 6 = -1A$$

Compare the solution by solving for the branch current using conventional means

Review Example 4.13